... "But look, you found the notice, didn't you?". "Yes," said Arthur, "yes I did. It was on display in the bottom of a locked filing cabinet stuck in a disused lavatory with a sign on the door saying 'Beware of the Leopard."

`The hitchhiker's guide to the (critical) planar Ising model.`

Basic notation:

- $V(G)$ – vertices, $F(G)$ – faces, $\diamondsuit(G)$ – edges, $E(G)$ – oriented edges of G;
- $\mathcal{E}(G)$ – set of all *even* (i.e., degrees of all vertices are even) subgraphs of G;
- edge weights $x_e = \exp(-2\beta J_e)$ (or $x_e = \tanh(\beta^* J_e^*)$), *parametrization* $x_e = \tan(\frac{1}{2}\theta_e)$.

$$Z(G) \;:=\; \textstyle\sum_{C\in\mathcal{E}(G)} x(C), \qquad x(C) := \prod_{e\in C} x_e.$$

- Graphs $G^{\mathrm{F}}, G^{\mathrm{K}}, G^{\mathrm{C}}$, double cover $\Upsilon^\times(G)$ and the graph G^{D}: TURN THE PAGE!
- $(x_{[u_1,\ldots,u_m]})_e := -x_e$ on edges crossing the (dual) paths linking faces $u_1,\ldots u_m$ pairwise;
- $\mathcal{E}^{[v_1,\ldots,v_n]}(G)$ – set of all subgraphs of G such that $v_1,\ldots,v_n\in V(G)$ have odd degrees and all other vertices are even;

$$Z^{[v_1,\ldots,v_n]}_{[u_1,\ldots,u_m]}(G) \;:=\; \textstyle\sum_{C\in\mathcal{E}^{[v_1,\ldots v_n]}(G)} x_{[u_1,\ldots,u_m]}(C).$$

Kadanoff–Ceva formalism:

- *spins* σ_u are assigned to faces of G, *disorders* μ_v – to vertices of G.
- *real-valued fermions* $\chi_c := \mu_{v_c}\sigma_{u(c)}$ are assigned to $\Upsilon(G)$ = corners of (faces of) G; $\Upsilon^\times(G)$ = the double-cover of $Y(G)$ ramified over each edge, vertex and face of G.
- Dirac spinor $\eta_c := \varsigma\cdot\exp[-\frac{i}{2}\arg(v(c)-u(c))]$, defined on $c\in\Upsilon^\times(G)$ (where $\varsigma\in\mathbb{C}: |\varsigma|=1$ is a fixed global prefactor, e.g. $\varsigma = e^{i\frac{\pi}{4}}$ or $\varsigma=1$);
- *(complex)-valued fermions* $\psi_c := \eta_c\chi_c$.
- For a collection ϖ of vertices $v_1,\ldots,v_{n-1}\in V(G)$ and faces $u_1,\ldots,u_{m-1}\in F(G)$, let

$$\mathcal{O}^{[\mu,\sigma]}_\varpi \;:=\; \mu_{v_1}\ldots\mu_{v_{n-1}}\sigma_{u_1}\ldots\sigma_{u_{m-1}}\,;$$

  $\Upsilon^\times_\varpi(G)$ be the double-cover of $\Upsilon(G)$ ramified over each edge, vertex and face of G except over the elements of ϖ and $\Upsilon_\varpi(G)$ be the double-cover ramified over ϖ.
- The real-valued observables $X_\varpi(c) := \mathbb{E}[\chi_c\mathcal{O}^{[\mu,\sigma]}_\varpi]$ are spinors on $\Upsilon^\times_\varpi(G)$; the (complex)-valued observables $\Psi_\varpi(c) := \mathbb{E}[\psi_c\mathcal{O}^{[\mu,\sigma]}_\varpi]\in\eta_c\mathbb{R}$ are spinors on $\Upsilon_\varpi(G)$.

Propagation equation for fermions: (see the TA session)

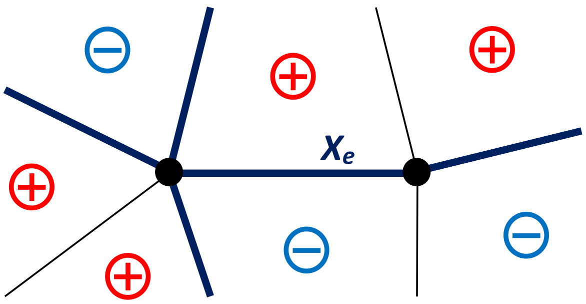
- $X_\varpi(c_2) = X_\varpi(c_1)\cdot\cos\theta_e + X_\varpi(c_3)\cdot\sin\theta_e$ for each triple of (consecutive on $\Upsilon^\times_\varpi(G)$ [!]) corners surrounding a quad (edge) $e\in\diamondsuit(G)$), where $\theta_e := 2\arctan x_e$.
- Smirnov's reformulation for the **critical Z-invariant Ising model on isoradial graphs (rhombic lattices):** for $z\in\diamondsuit(G)$, there exists $\Psi_\varpi(z)\in\mathbb{C}$ such that

$$\Psi_\varpi(c) \;=\; \tfrac{1}{2}[\,\Psi_\varpi(z)+\eta_c^2\cdot\overline{\Psi_\varpi(z)}\,] =: \mathrm{Proj}[\Psi_\varpi(z);\eta_c\mathbb{R}].$$

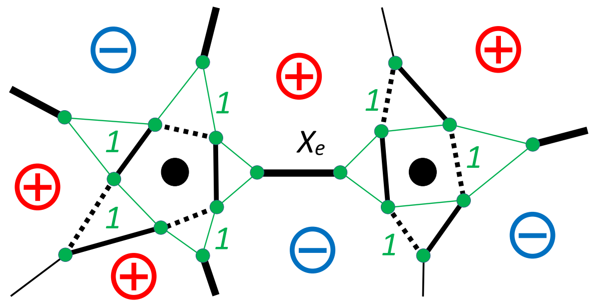
Graphs constructed from a given planar graph G:

- vertices of G^{F} $\leftrightarrow$ oriented edges and corners of G;
- vertices of G^{K} $\leftrightarrow$ oriented edges of G;
- vertices of G^{C} $\leftrightarrow$ corners of G;
- vertices of $\Upsilon^{\times}(G)$ $\leftrightarrow$ *double cover* of corners of G;
- G^{D}: bipartite, two vertices (of different colors) per corner of G.

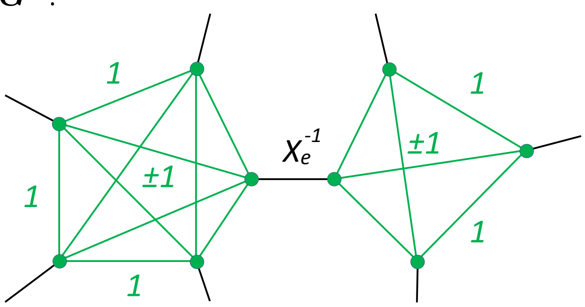
$\boldsymbol{G}$:



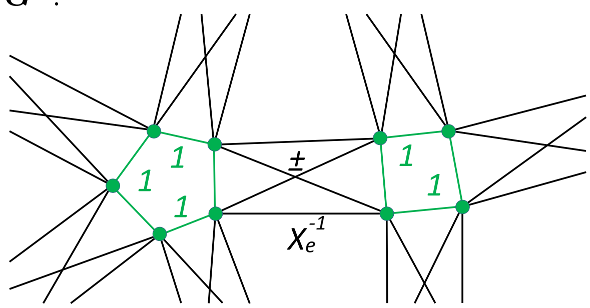
$\boldsymbol{G^{\mathrm{F}}}$:



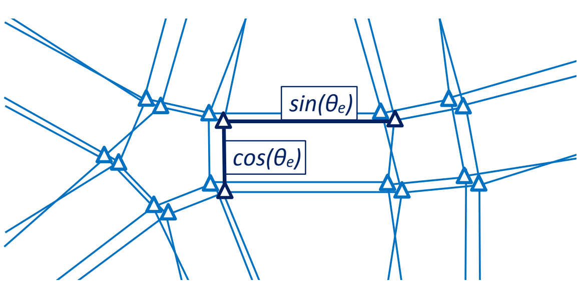
$\boldsymbol{G^{\mathrm{K}}}$:



$\boldsymbol{G^{\mathrm{C}}}$:



$\boldsymbol{\Upsilon^{\times}(G)}$:



$\boldsymbol{G^{\mathrm{D}}}$: